

Parameterized Meshes

Lecture 22

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Fri, Oct 13, 2017

- 1 Parameterized Surfaces
 - A Parameterized Sphere
 - A Parameterized Paraboloid
- 2 Assignment

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Surfaces Defined by Functions

- Let $F(x, y, z) = 0$ be an equation that implicitly defines a 2-dimensional surface in 3-dimensional space.
- For example,
 - $x^2 + y^2 + z^2 - 1 = 0$ defines a sphere.
 - $x^2 + z^2 - y^2 = 0$ defines a cone.

Parameterizing the Surface

- Let x , y , and z be given in terms of **parameters** s and t , with $a \leq s \leq b$, $c \leq t \leq d$.
 - $x = x(s, t)$
 - $y = y(s, t)$
 - $z = z(s, t)$
- Typically, $[a, b]$ and $[c, d]$ are $[0, 1]$ or $[0, 2\pi]$.
- Then $P(s, t) = (x, y, z)$ is a point on the surface.

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- Latitude and longitude is a good example on the surface of a sphere.

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A Sphere of Radius 1

Example (A Sphere of Radius 1)

- Use spherical coordinates, where
 - s is the angle around the vertical axis through the poles (longitude),
 - t is the angle up from the equator (latitude).
- Then let

$$x = \cos t \sin s$$

$$y = \sin t$$

$$z = \cos t \cos s,$$

with $0 \leq s \leq 2\pi$ and $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$.

A Sphere of Radius 1

Example (A Sphere of Radius 1)

- Then

$$P(s, t) = (\cos t \sin s, \sin t, \cos t \cos s)$$

is a point on the surface.

- Check:

$$x^2 + y^2 + z^2 = (\cos t \sin s)^2 + (\sin t)^2 + (\cos t \cos s)^2$$

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A Paraboloid

Example (A Paraboloid)

- s represents an angle around the central axis.
- t represents the radius of a circular horizontal cross-section.
- The paraboloid $y = x^2 + z^2$ of height 1 is defined by

$$x = t \sin s$$

$$y = t^2$$

$$z = t \cos s$$

with $0 \leq s \leq 2\pi$ and $0 \leq t \leq 1$.

A Paraboloid

Example (A Paraboloid)

- Then

$$P(s, t) = (t \sin s, t^2, t \cos s)$$

is a point on the surface of the paraboloid.

- Check:

$$x^2 + z^2 = t^2 \sin^2 s + t^2 \cos^2 s$$

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- Check:

$$\begin{aligned}x^2 + z^2 &= t^2 \sin^2 s + t^2 \cos^2 s \\&= t^2 (\sin^2 s + \cos^2 s) \\&= t^2 \\&= y.\end{aligned}$$

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- Assignment 18.